Introduction to quantum physics for philosophers

by Sylvain Poirier

Philosophical lessons from mathematics & physics

From the study of the foundations of mathematics:

- Mathematics has a "growing block flow of time":
  the "past" of any discourse exists but its "future", out of sight, is unpredictable.
- Mathematics is necessary: all possibilities are equally real (once “verified”)

Laws of physics logically require paradoxical features:

- To describe physical time and contingency by necessary mathematics (actually timeless ones)
- Causes of an effect must have finite complexity (amount of information) for laws to be definite

Relativity theory

- Describes time as a geometric dimension (eternalism)
- Implies locality of causes (speed of light limits information)
- But involves continuous space : locally infinite information ?

Quantum theory and philosophy

Quantum theory achieves paradoxical solutions :

- Expresses indeterministic laws by deterministic mathematics
- Explains local finiteness of information in a continuous space

Its key: it only describes probabilities, no reality !
Space continuity : continuous evolution of probabilities of finitely complex states

Problem : **Probabilities of what** ????

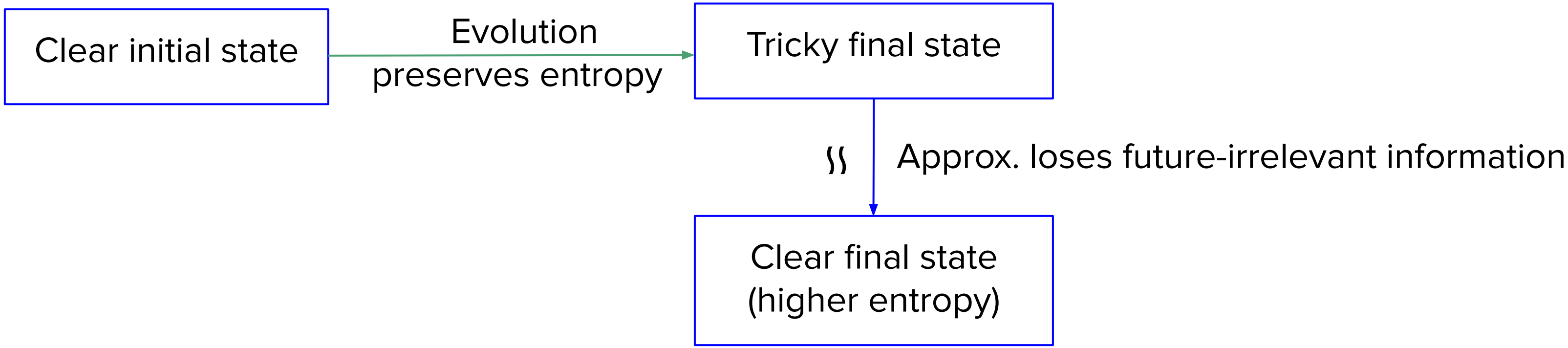
Distinct possibilities, definition of measurements, randomness (actualization of possibilities), needed to give meaning to probabilities, are absent from the theory
→ Requires an interpretation.

Choice of interpretation = choice of ontology (what is real and how)

Irreversibility explained

Fundamental laws are time-symmetric.

Entropy creation (irreversibility) emerges in macroscopic approximation, by *assuming some clear initial state* : a time-asymmetric assumption beyond the laws.

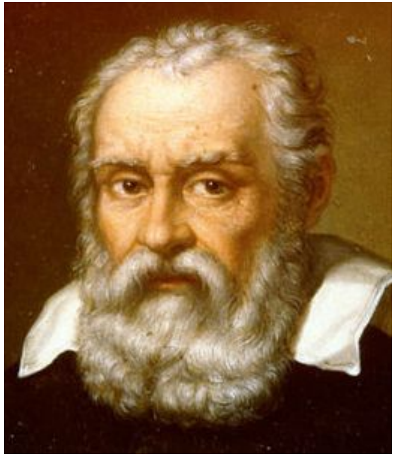


States, with time-symmetric evolution law, are “probabilistic” and must be *interpreted as* giving relevant probabilities for predictions, not for retrodictions !
Reverse evolution on clear final states would give very wrong “probabilities” for retrodictions.

Plato :

"Let no one ignorant of geometry enter"

Galileo Galilei :



“Philosophy is written in this grand book — I mean the universe — which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures, without which it is humanly impossible to understand a single word of it; without these, one is wandering about in a dark labyrinth.”

Classical probabilistic evolution

(Markov chains)

Classical probabilities = probabilities ***of something***

- Probabilistic states
- Probabilistic evolution laws
- Probabilistic measurement
- Correlations

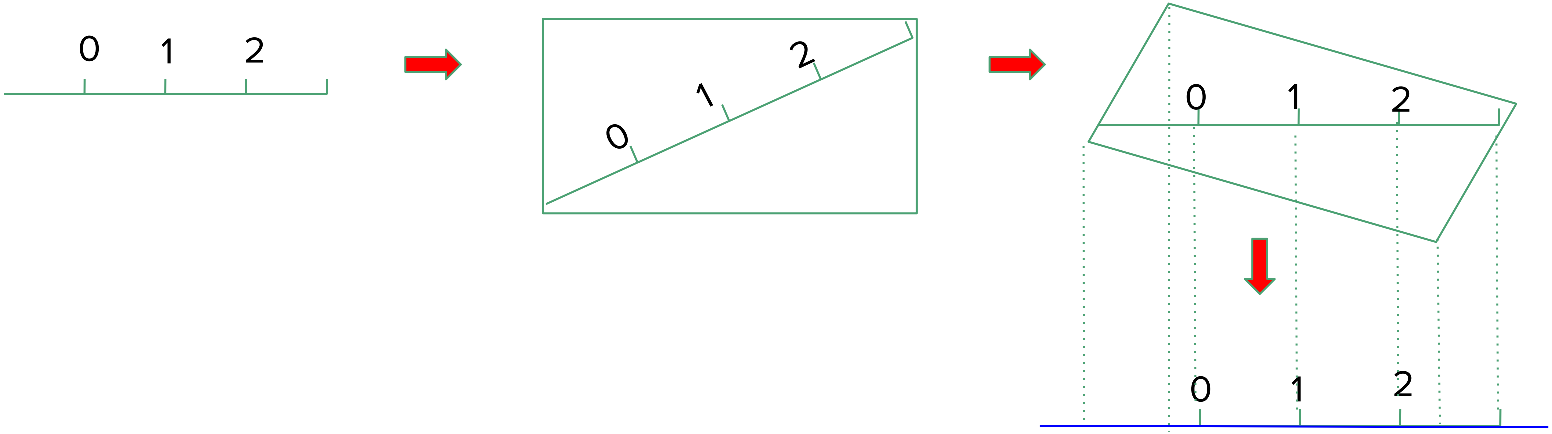
to be expressed by the mathematical language of affine geometry

Affine geometry

Poorer than Euclidean geometry: fewer structures (concepts), more transformations (beyond rotations & symmetries):

List of structures : Straight lines (alignment), parallels, parallelograms, middles and other barycenters

These are preserved by *affine functions* between spaces with different dimensions:



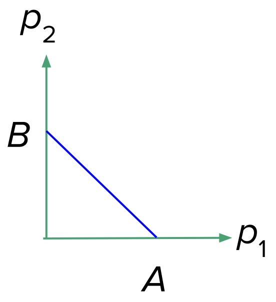
Classical probabilistic state

Consider a physical system with a list of n possible elementary states A,B,C....

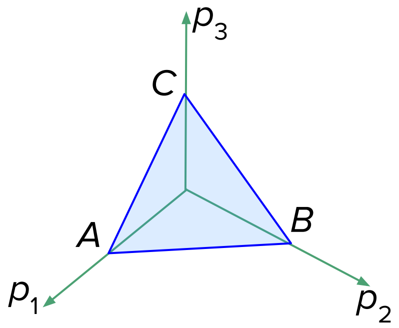
A probabilistic state = the data of their probabilities p_1,...p_n: real numbers such that

$$p_1 \geq 0, \ldots, p_n \geq 0 \qquad\qquad p_1 + .. + p_n = 1$$

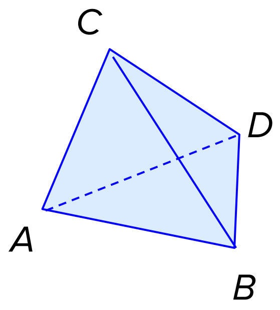
The set of all probabilistic states over n elementary states is a simplex with dimension $n-1$



n=2 : a segment



n=3 : a triangle



n=4 : a tetrahedron

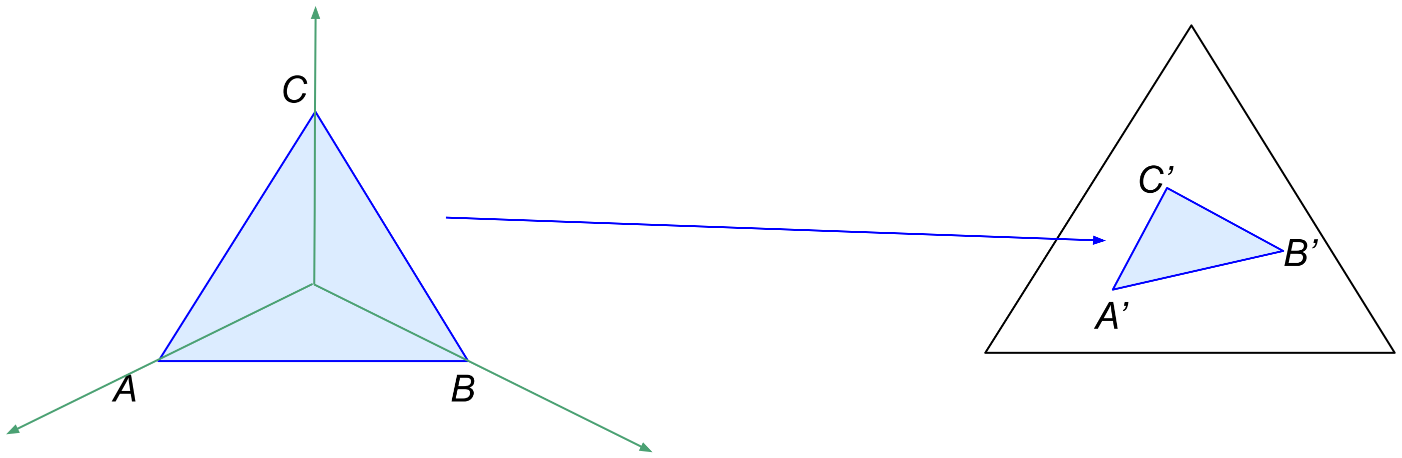
Pure states = states with entropy 0 = elementary states

For n=3 : vertices A=(1,0,0), B=(0,1,0), C=(0,0,1)

Other states (inside) have non-zero entropy.

Probabilistic evolution

Elementary states *A*,*B*,*C* evolve into respective probabilistic states *A’*,*B’*,*C’*.
This determines the evolution of a probabilistic state into another, represented by the unique affine function extending ($A \mapsto A'$, $B \mapsto B'$, $C \mapsto C'$): affine functions are those which preserve barycenters.
The concept of *barycenter* of a list of points with coefficients, commonly used to give the center of gravity of a system of masses at different positions, here gives the probabilistic state obtained as a “superposition” of a list of states possibly produced by a given process, weighted by the respective probabilities of producing them.



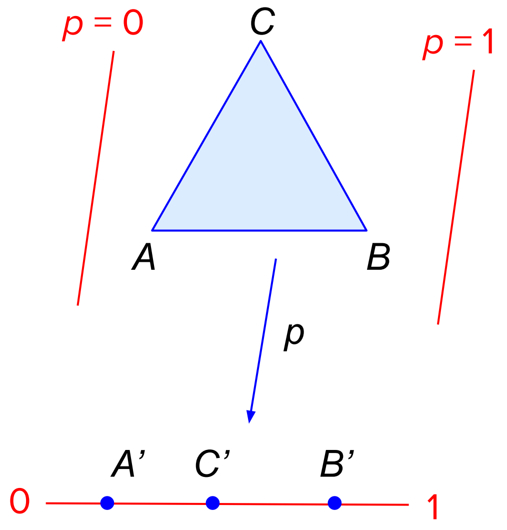
Measurements

A measurement is an evolution to a system with elementary states classified in two groups :
0 = No ; 1 = Yes.

Summing up the final state as probabilistic between two final elementary states "0" and "1", reduces this to an affine function p into the segment [0,1].

This measurement p can be represented by two hyperplanes (p = 0, p = 1) containing the simplex.

Hyperplane = subspace with dimension (dim of containing space − 1)

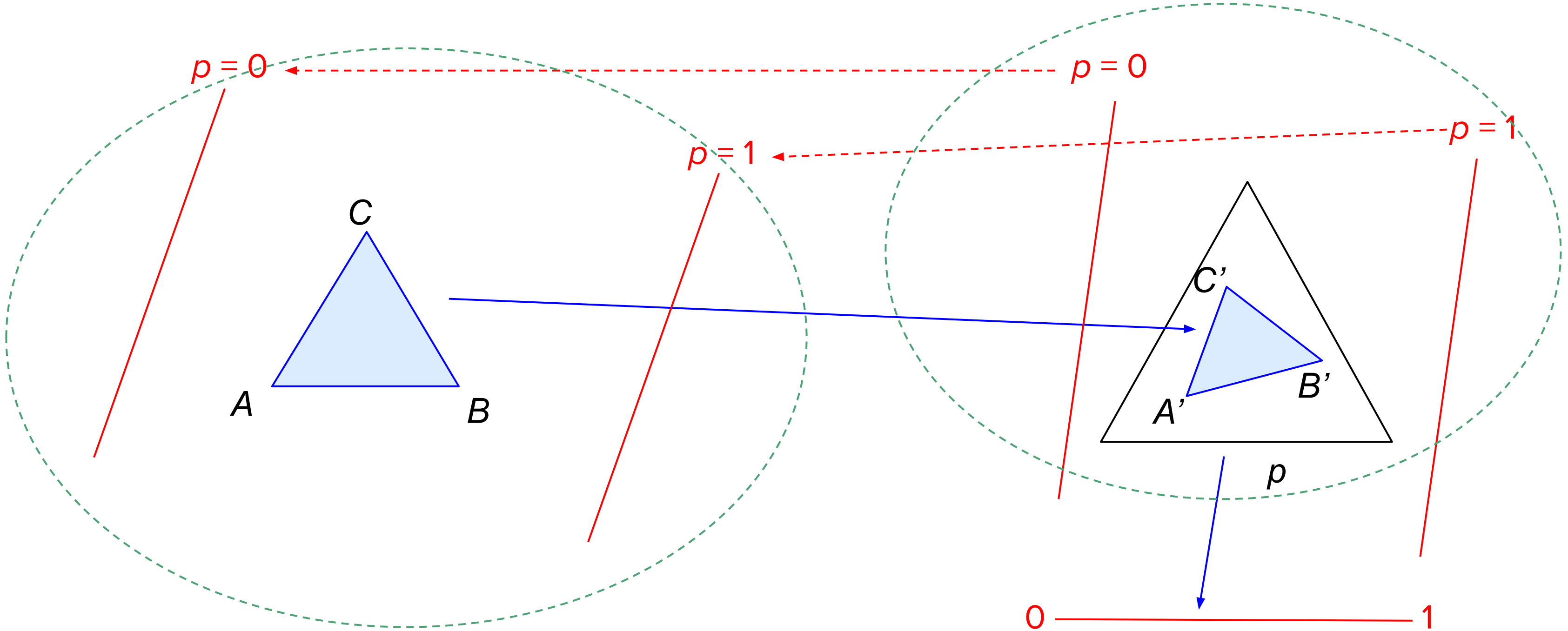


Its image = probability of result "1".

If p is a constant function, the equations p = 0 and p = 1 do not define hyperplanes : this is a *non-measurement*.

Retro-evolution of measurements

Measurements applied after evolution are interpreted as measurements of the initial system



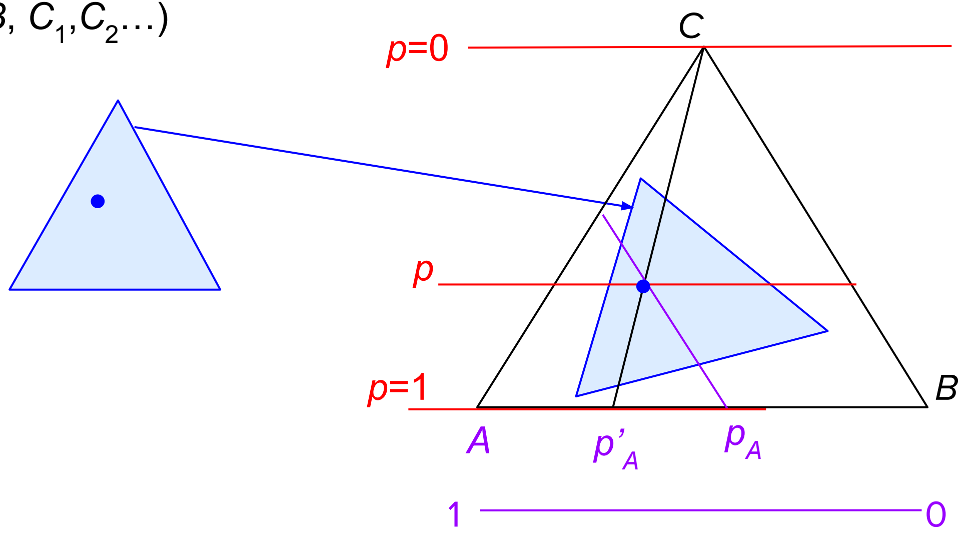
Effects of measurements

Measurement process : (some state) → (A, B, C_1,C_2…)
Observed result “Yes” = (A or B)
C sums up all “No” states

p_A = probability of A ignoring the result
$p = p_A + p_B$ = probability of “Yes”

p'_A = probability of A knowing the “Yes” result

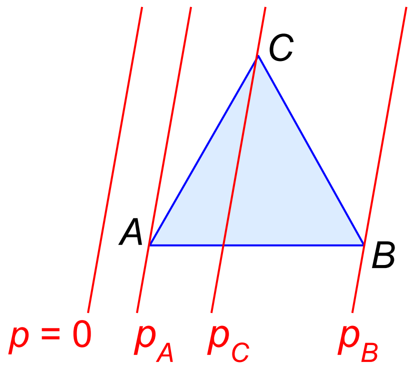
$p'_A = p_A/p$ $\qquad$ $p'_B = p_B/p$

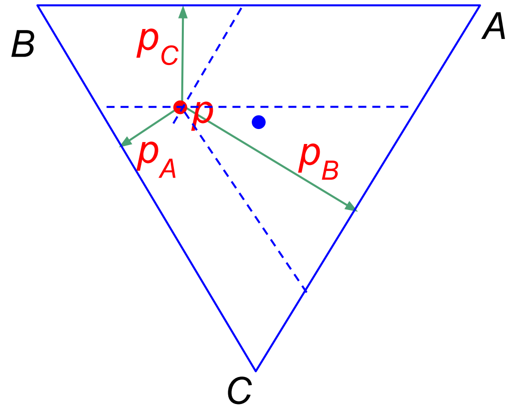


New probabilistic state (p'_A, p'_B) where $p'_A + p'_B = 1$ relative to the measurement result “Yes”
Initial state → (p_A, p_B, p_C) → (p'_A, p'_B) = final state
A measurement result acts as a projective transformation sending (p=0) to infinity
(division by the probability p of this result)

Dual simplex of all measures

The information given on the state of a system, by a given measure ("yes" result) with a probabilities function p defined by its (positive) coordinates p_A, p_B, p_C … (probabilities of "yes" for elementary states A, B, C…), only depends on the ratios between p_A , p_B , p_C …
Dividing p by (p_A+p_B+…= n × prob. of "yes" on the equiprobable state) gives a representation of p by a point in the *dual simplex* : the set of "all possible measures", also a simplex. Its vertices ("pure measures") are those giving yes with nonzero probability to only one pure state.
This belongs to a projective space, figured as affine by reference to the equiprobable state (center of the simplex of states).

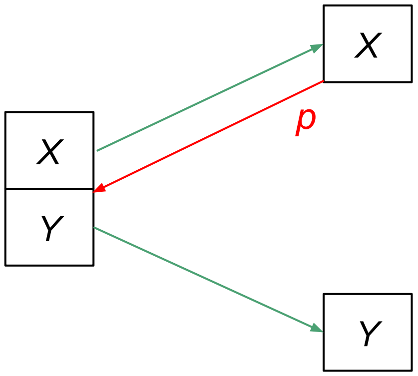




As the evolution of states is affine, the retro-evolution of measures appears as a projective function between dual simplexes which preserves their centers (non-measures).

(evolution preserves equiprobability) ⇔ (retro-evolution looks affine)

Correlations



System of two components X, Y initially connected, then going apart.
Its number of pure states is the product $m \times n$ of those of X and Y.

Its probabilistic state S is *correlated* if some measurement result on X with proba function p has an epistemological effect on the state of Y (and equivalently vice-versa), going by retro-evolution of p from the final X to the initially connected system, then forward evolution from it to the final Y.

Equivalently, S is correlated if its entropy is lower than the sum of entropies of the state of each component relative to the non-measurement of the other.
A maximal correlation for $m = n$ is given as equiprobable between the n pure states (A_X,A_Y), (B_X,B_Y)... of a chosen bijection between the pure states of X and Y. It gives the copy of p as state of Y.
Its entropy is $\ln(n)$ while those of each component are also $\ln(n)$, with sum $2\ln(n)$.

The final state of Y relative to p is, as previously seen, a projective function of S for fixed p ; but it is also a projective function of p for fixed S (which looks affine if the final state of X for this S with non-measured Y is the equiprobable one).

Quantum theory

Cling to the brush, I remove the ladder !

A theory of probabilities ***of nothing but*** results of undefined measurements

- Quantum states
- Quantum evolution
- Quantum measurement
- Quantum entanglement

expressed by the same language of affine geometry

Quantum states

Any physical system with given limits of available space and energy, has a finite *number of states* (non-zero natural number) n. The shape of its set of possible quantum states, is determined by n as a convex region of an (n^2-1)-dimensional affine space (not a simplex).

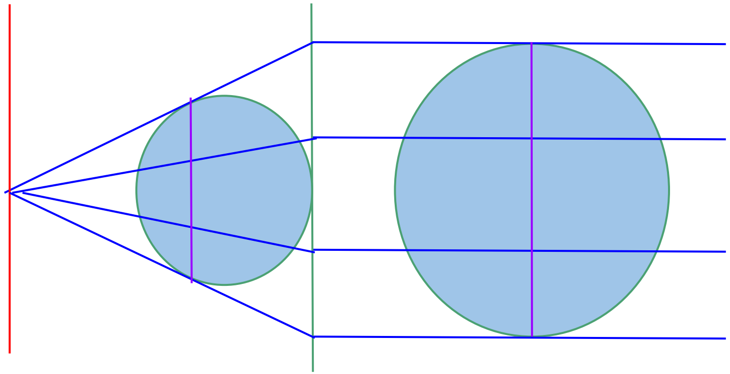
A 2-states quantum system is called a *qubit*.
Its set of possible states is **a ball** in a 3D space.
Its surface (sphere) is its set of *pure states* = with entropy 0.
Only antipodal pure states can be “clearly distinct” from each other.
The entropy of a qubit only reaches is maximum ln(2) in the center.
Inside points are classical superpositions (barycenters) of 2 pure states in many ways (intersections of the sphere by any line through this point).

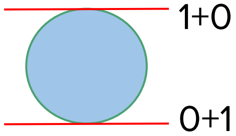
Generally, the set of pure states of an n-states system is a $(2n-2)$-dimensional “surface”; any inside point is, in many ways, a barycenter of n of them (no less) with positive coefficients. Each border point is inside of a unique set of states which can be seen as that of a k-states system for some $k < n$, by ignoring the rest of possible states. Pure states are those for which k=1.

Quantum measurement

Much works like in the classical case: a measurement is figured by a list of affine functions from the set of possible states into [0,1], whose sum is constant =1, giving the probabilities of each possible result on each possible state of the system. Looking at each result for the information it gives on the system if obtained, sums up this affine function to its hyperplane of zero values, also figured as a point in the dual set with the same shape as that of the set of states (no more a simplex); the new state relative to a given measurement result is a projective function of the initial state which sends this hyperplane to infinity.

However these projective functions can no more preserve all pure states. They can send all pure states to pure states, but not all projective functions from a set of states into another can be so realized. For a qubit, any projective transformation of the sphere of pure states to itself is possible only if it preserves the orientation (mirror symmetry would squeeze the radius by 3, leaving no state pure).





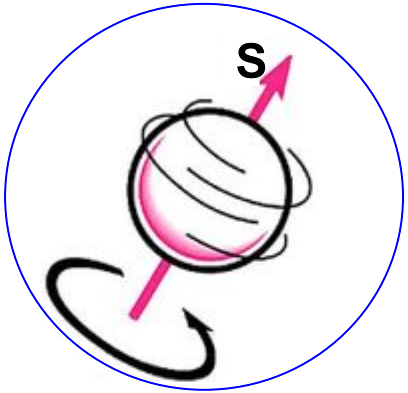
Standard measurements of qubits are pairs of antipodal pure measures. They destroy all other information on its state.

States in phase space

Classical mechanics describes states of systems as points evolving in a *phase space*, geometric space with even dimension. Each position coordinate gives a pair of 2 dimensions of the phase space: position and momentum, called *conjugate* of each other. Areas along a pair of conjugate dimensions are *amounts of action* = the same kind of quantity as angular momenta (= momentum × length), and as (energy × time). So, d position coordinates give a $2d$-dimensional phase space whose volumes are d-th powers of actions. Isolated systems evolve by transformations of the phase space which preserve volumes.

Quantum mechanics introduces the Planck constant $h = 2\pi\hbar$ as a fundamental unit of action. Classical approximations of quantum n-states systems correspond to phase space regions with volume $n\ .\ h^d$ (that is n taking h as unit).
Choosing n “clearly distinct” pure states of an n-states system, roughly means cutting the phase space into n pieces with volume h^d each.

Classical spin

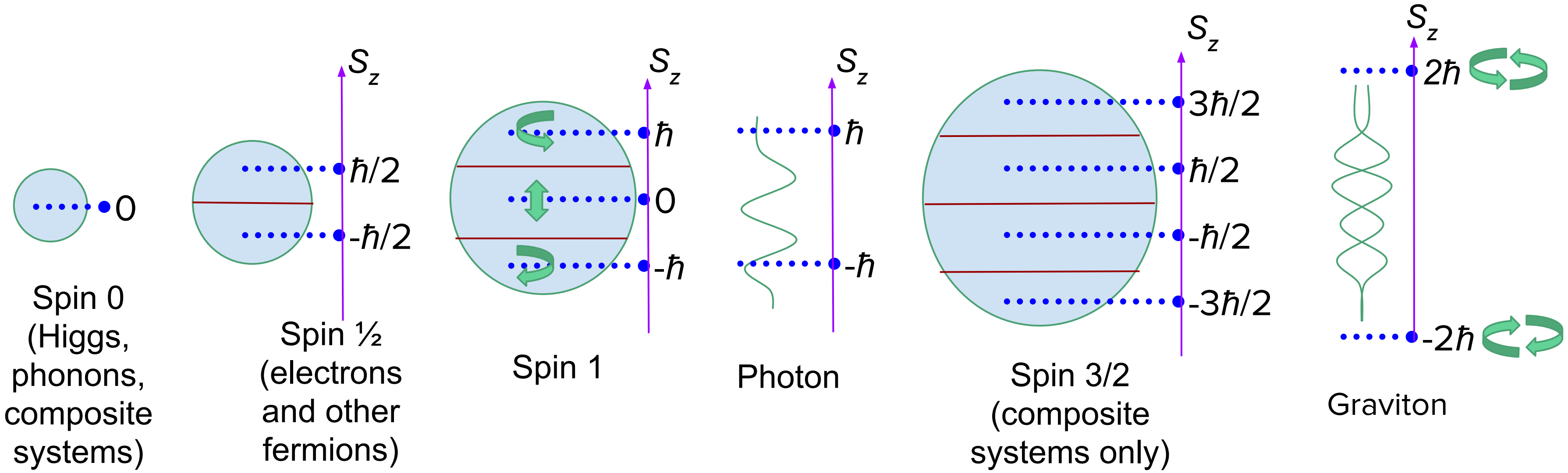


The spin of a particle is a quantum property also describable by classical mechanics.
Particles should be imagined as spinning tops, but they have no detail (angular position) actually rotating at any speed, and their spinning can neither stop nor accelerate. They only have an angular momentum with respect to their own position. It is described by a vector **S** along the axis, with fixed norm *S*.

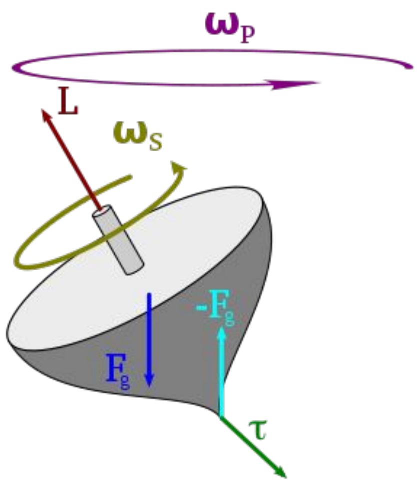
This **S** can evolve along the set of such vectors: a sphere with radius *S*.
In lack of rotational position coordinates, this sphere alone is the phase space of spin states, with its 2 dimensions serving as the conjugate of each other (d=1).
While spheres with radius r have area $4\pi r^2$, the special measure of area in this phase space gives its area a value (amount of action) of $4\pi S$.

Quantum spin states

The state of spin of a particle can be exactly measured along one chosen axis (but then not along other axis). Possible values of the angular momentum along this axis then differ from each other by multiples of $\hbar$, and there is only one possible state per value. Indeed such values fit a division of the phase space into regions with each an area of h :



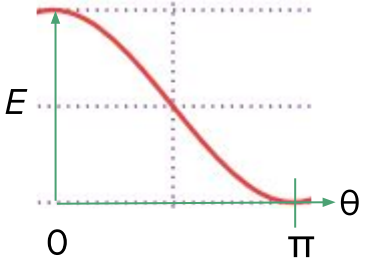
Quantum evolution



Quantum states can evolve by diverse affine transformations if interactions let them “lose” information into the environment (not observed), but only by rotations otherwise. The spin of the electron (½), which gives it a magnetic dipole moment, rotates as given by the magnetic field when approximating this field as classical (neglecting emission / absorption of photons).

This rotation is explained in the same way as the precession of the spinning top under gravity: an energy difference E between “top” and “bottom” states “pushes” the angle θ of the spin direction with the vertical, by a torque reaching at the equator $\theta = \pi/2$ its maximum $E/2$ = “speed” of horizontal variation of the angular momentum vector with norm $\hbar/2$. This gives an angular speed of precession $E/\hbar$, thus a frequency E/h.

As any rotating magnetic dipole emits an electromagnetic wave, the rotating spins of electrons can emit photons with this frequency.



This evolution law is actually valid for all qubits: the energy function is affine with a difference E between its maximum and minimum points, and the state rotates with angular speed $E/\hbar$ around them.

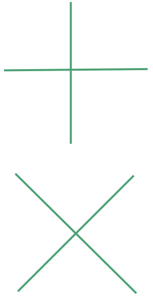
Quantum entanglement

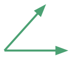
Entanglement is the quantum version of correlation : the language of quantum theory cannot distinguish both concepts ; neither can be used for communication. As a pair of electrons is a spin 0 system, the spins of both electrons are entangled : the angular momentum, sum of those from the spins of both electrons, is 0 along any chosen axis. Once spatially separated, if not disturbed, their spin are so correlated that the state of the one with respect to a measure of the other, is a copy of it (as a point in the dual set), with change of sign (in intuitive conventions): a measure of one spin which would have given probability 0 to a given state, puts the spin of the other in that state. Classically correlated states could not be pure, but this state of entanglement is pure, with entropy 0, while each spin taken separately appears undetermined, with entropies ln(2) + ln(2).

All pure entangled states of a 4-states system made of 2 qubits, can be described as projective transformations from the sphere of measures of one qubit, to the sphere of states of the other (thus preserving purity), still with opposite orientation as above.

Quantum non-locality

While correlations could be called an “epistemological effect”, quantum entanglement fails to be explainable by hidden local causes created in the initial bound system, as any correlation satisfies inequalities (Bell’s inequalities) which entanglement violates; this is called *quantum nonlocality* or “*spooky action at a distance*”. Here is the simplest illustration.

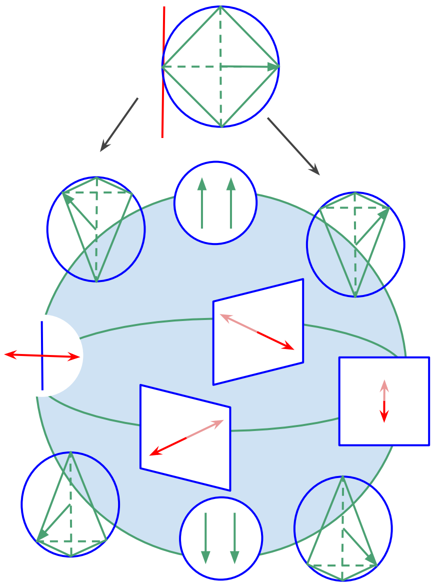


Taking an entangled pair of qubits, let one experimenter choose to measure one qubit either horizontally or vertically, while the other measures the other qubit (rotated by 180° to make states “equal” for clarity) along either of both diagonals. Then their probability to find “neighbor” results like  is about 85%, vs. 15% to find “opposite” ones like 

But classical correlations do not allow better average probabilities for this than 75% vs 25% : from a given result on one edge, one only needs to switch 4 times to a neighbor vertex, each time losing the small fraction of disagreement cases, to reach the opposite result.
This may be seen as a “proof” of non-local causality or not, depending on the interpretation.

Quantum decoherence

Measurements can be understood as a macroscopic view over evolution processes. Standard measurements of qubits duplicate their state along the given axis, by evolution into a qubit of entangled states of the system with the measurement device. Pure states which were orthogonal to this axis become states of maximal entanglement, where states of components correspond by mirror symmetries with respect to planes containing the axis.

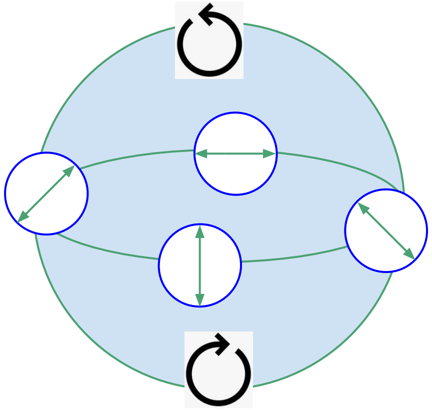


The copy (state of the measurement device) is then duplicated again many times in the same way along the same axis. As soon as one copy is lost into the environment (entropy creation process), all states, now relative to the non-measurement of the lost copy, are projected onto their vertical axis, reducing the description to that of classical states and correlations.

States of the photon

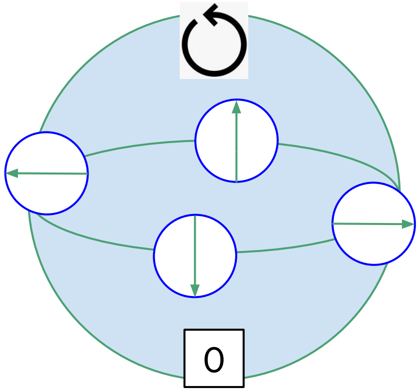
Consider 3 possible clearly distinct states, or aspects of a state, of the electromagnetic field :

- Absence of any photon.
- One photon with angular momentum +*ħ* / propagation axis
- One photon with angular momentum -*ħ*.



Any pair of them forms a qubit, with other possible states:

Possible presence of a circularly polarized photon: the electric field rotates around the propagation axis at the frequency of the photon's energy

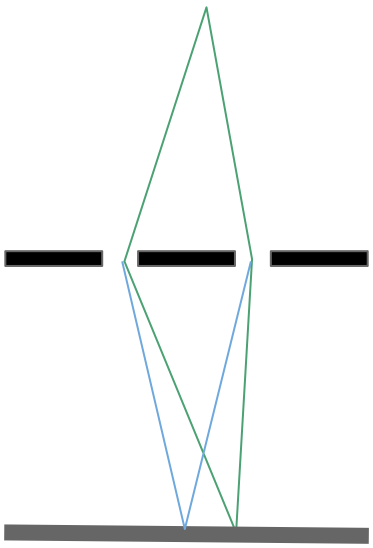


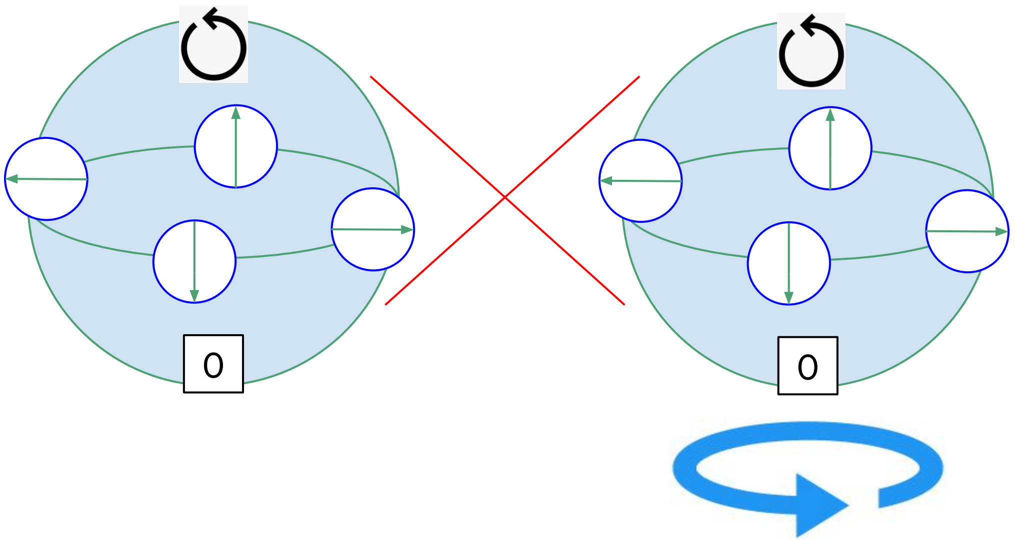
Poles (+/- *ħ*) : circular polarization
Equator: linear polarization

In spin 1 massive particles,
linear polarizations from (+/- *ħ*) qubit
= 0 momentum / horizontal axis

Double-slit experiment

A photon going through a screen with 2 slits and then hitting another screen, its probabilities of detection undergo interferences as each detection point measures an entangled pair of states of the electromagnetic field between both slits, with diverse phase differences (= states of this pair)





Interpretations of quantum theory

A metaphysical debate

List of interpretations

Interpretations of quantum theory fall into 4 main classes (without even perfect consensus on this list), giving different answers on ontology, causality and locality. They can be sorted by their answers on the time of “wave-function collapse” = when the result of a measurement gets determined, with respect to given intuitively described circumstances. A fifth one which nobody supports (?) can be added for the sake of the discussion :

Time of wave-function collapse	**Interpretation**
Birth of the universe	Hidden variables (Bohm) - superdeterminism
Measurement	Spontaneous collapse (GRW…)
First conscious perception	Mind makes collapse (Von Neumann-Wigner)
After first conscious perception	Slow collapse / solipsism (Wigner’s friend exp.)
Never	Many-worlds (Everett)

Spontaneous collapse (= Objective collapse)

To realize the random change of state by measurement requires to modify the evolution law (difficult problem, first proposal Ghirardi–Rimini–Weber 1985) only “after decoherence” which is an emergently defined condition. How long after ?
Risk of happening before full decoherence, leads to observable effect = chance of discovering the mechanism (still never observed).
To avoid breaking conservation laws (a contradiction in GR), collapse remains incomplete (other results keep a small probability).

Reality = quantum state
Causality = probabilistic
Non-local. Thus breaking relativistic invariance (unless admitting retro-causality ?)
Problem: conspirational aspect of yet unobserved mechanism

Mind makes collapse

(Eddington 1928, J. von Neumann 1932, London & Bauer 1939, Eugene Wigner 1960s…)

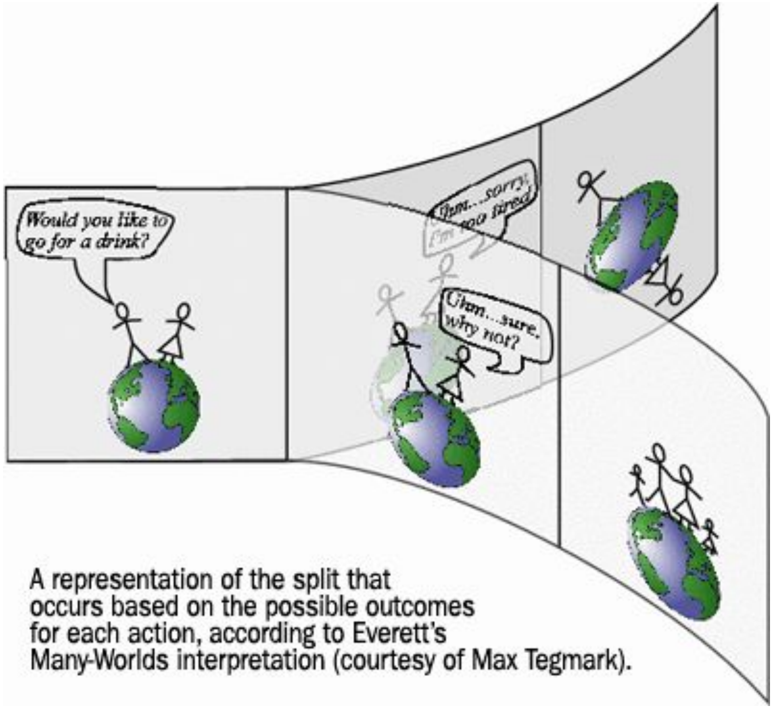
Conscious perception selects one result to make it real.
Solves the “interaction problem” of dualism in philosophy.
Non-physicality allows to put decoherence as a condition for collapse.
Variant by Seth (J.Roberts): several possibilities can be chosen in parallel.

Reality : conscious perception (non-physical)
Physical reality (emerges): quantum state from all past perceptions.
Causality : free will can operate by not following physical “probabilities”.
Locality ? unclear, beyond “physics”.
Relativistic invariance *of physics* is respected.

Many-worlds (Hugh Everett, 1957)

“the bare theory”

Quantum evolution is the only law. Measurement is an appearance emerging by decoherence.
All possible “measurement results” coexist in parallel reality branches.
“Probabilities” are interpreted as shares of “existence” seen as a divisible quantity.
They cannot be explained as ratios between branches numbers.
Measurement and branching are local (relativistic invariant). “Non-locality” is mere appearance from later matchmaking between local branches.
Relatively to oneself, the state of the rest of the universe is very undetermined.



A representation of the split that occurs based on the possible outcomes for each action, according to Everett's Many-Worlds interpretation (courtesy of Max Tegmark).

Hidden variables

Behind the Many-worlds quantum state (which must exist anyway), another kind of state decides which world (branch) is “real”.

Simple proposal (de Broglie-Bohm theory): exact position coordinates.

Infinite complexity, both from the infinitely large (non-local states and evolution law) and the infinitely small.

The theory has different versions ; usually no hidden variable for spin.

Only defined for non-relativistic case (no emission/absorption of photon).

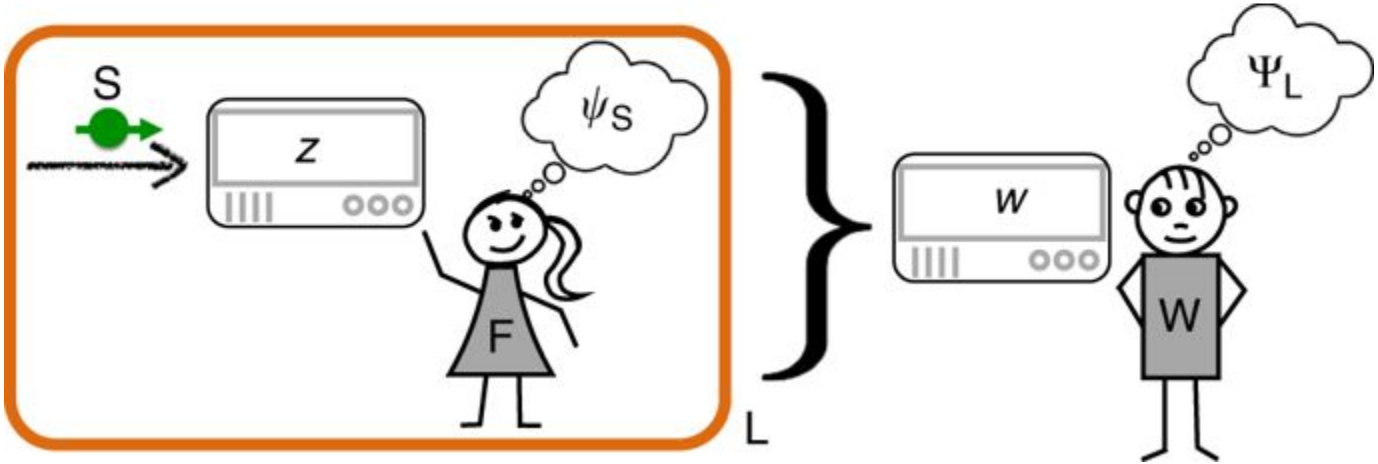
Version compatible with QFT still unknown.

Requires an absolute time (cannot be simply introduced in GR).

Causality: determinism from initial values (yet unexplained).

Reality = hidden variables which follow one branch of quantum state; can switch between branches (same position, different momentum).

Wigner's friend experiment = Solipsistic interpretation



May be taken as relative solipsism = quantum Bayesianism (QBism)
Ignores ontology, focuses on the epistemological aspect.

Slow collapse

Same physical theory as spontaneous collapse, but slower.
Observers are first in superposition like with many-worlds.
Later, only one branch of reality survives.

Question for supporters of spontaneous collapse interpretation:

What is wrong with slow collapse ?

About myself

- Explored maths & physics in my free time
  (General Relativity in high school)
- Mathematics PhD... 1 year university teaching.
- Left all academic duties which were a waste of time.
- Focus on writing clean, optimized expressions of the foundations of mathematics and physics (for free)
  Web site : **http://settheory.net**
- Includes philosophical aspects, inspired by mathematics (not by philosophers !)
- Other philosophical writings